MATHEMATICAL MODEL FOR THE DIAGNOSTICS OF VEHICLE HYDRAULIC BRAKING SYSTEM LEAKAGE

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Summary

The purpose of the diagnostic tests is to specify the technical condition of an object or process at a moment deemed as important. This specification is necessary in order to compare the actual momentary condition with the model condition, make a judgement on the applicability or inapplicability of the technical object.

The publication includes the technical and structural characteristics of the hydraulic braking system (HSUH). The mathematical model of the system's leakage diagnostics process was developed on the basis of the laws of logic and the set theory, including: front axis brake master cylinder chamber (KP_{HP}), right hydraulic sub-circuit (outside the brake master cylinder) of front brakes (PP_{HP}), left hydraulic sub-circuit (outside the brake master cylinder) of front brakes (LP_{HP}), rear axis brake master cylinder chamber(KP_{HT}), right hydraulic sub-circuit (outside the brake master cylinder) of front brakes (LP_{HP}), rear axis brake master cylinder), left hydraulic sub-circuit (outside the brake master cylinder) of rear brakes (PP_{HT}), left hydraulic sub-circuit (outside the brake master cylinder) of rear brakes (PP_{HT}).

Key words: brake system, hydraulic circulation, double-sequence brake master cylinder, leakage, modeling

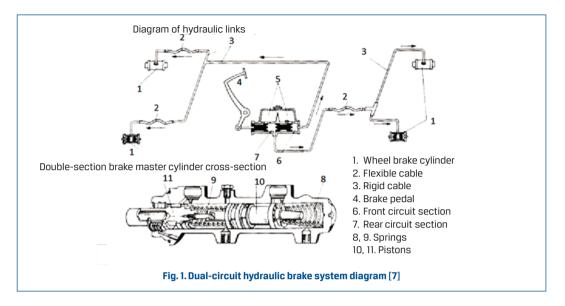
1. Introduction

The hydraulic brake system uses the Pascal's law stating that pressure induced from the exterior to a liquid's surface is distributed in it equally in all directions.

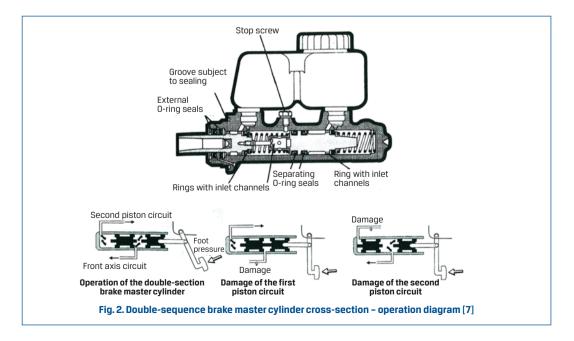
The dual-circuit hydraulic brake system (fig. 1) is currently the most commonly used in the equipment of passenger vehicles. It differs from the single-circuit system in terms of the double-sequence brake master cylinder.

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Dual operation of the master cylinder (fig. 2) is obtained by using an additional piston, the so called free piston, which divided the cylinder interior to two working chambers. The expansion tank is located above the brake master cylinder and contains the fluid which can flow into the cylinder at both sides of the free piston via separate supply channels. Such structure allows for development of two circuits which activate the braking mechanisms. In case of damage to one of them, it is possible to stop the vehicle with the use of the second circuit.



2. Diagnostic parameters and technical condition evaluation criteria

The diagnostic parameters which specify the brake system's technical condition can be divided in the following groups [1, 2, 3, 4, 5, 11, 16, 17, 18, 19, 20, 21]:

- 1) parameters characterising the mechanism which activates the brakes,
- 2) parameters specifying the technical condition of braking mechanisms,
- 3) parameters characterising the effectiveness of the brake system's operation.

For brake systems with hydraulic activation, the technical applicability conditions can be specified as follows [1, 7, 8, 9, 16, 19, 20]:

- 1. system completeness the system should be completed in accordance with the vehicle's technical documentation.
- correctness of the system's element mounting in practice, this regards to the correctness of mounting of the brake master cylinder, brake lines and brake mechanism discs,
- 3. correct external condition of the system's elements confirmation of lack of mechanical damage (indentation, line abrasion, cracks, brake fluid leakage, etc.),
- 4. correct brake fluid level in the tank,
- 5. lack of air lock in the hydraulic circuit,
- 6. hydraulic circuit tightness
- 7. correct value of the brake pedal free-play,
- 8. correct value of the pedal reserve,
- 9. proper free-play in the braking mechanisms evaluated indirectly on the basis of the pedal reserve value or directly by means of a measurement,
- 10. correct values of the brake system operation effectiveness evaluation parameters:
 - a. maximum (or average) brake delay,
 - b. total brake force and supplementary parameters, which specify the dependence of changes in the brake force on particular wheels from the pressure on the brake pedal and the distribution of the brake force on the vehicle's sides and axles.

Evaluation of the maintenance of specified conditions (except the last one) is performed at the stage of initial diagnostics of the brake system. The first activity of the brake system's external inspection is checking its tightness. The simplest method of identifying leakage is to regularly check the level of brake liquid in the tank. The level of liquid should be within the two lines marked as MIN and MAX and cannot drop too quickly during the exploitation of the vehicle. In case of disc brakes it is acceptable for the liquid level in the tank to gradually become lower, as it is a normal phenomenon related to gradual degradation of the brake pad lining. Dual-circuit brake systems include two separate brake fluid tanks or a single tank divided into two chambers. When checking the brake fluid level in these tanks it is necessary to pay attention whether the fluid levels are equal in both brake circuits. The manufacturers of some vehicles mount devices in tanks, which constantly measure the brake fluid level, the excessive decrease of which is specified on the indicator panel. In case of identifying excessive decrease of the brake fluid in the tank, it is necessary to identify the leak. Leaks are identified by dark, damp stains visible on the surfaces of brake lines, their threaded connections and the body of the brake master cylinder. Leakage of the

brake master cylinder is revealed by the leakage of fluid from under the brake drum on the tyre on the internal side of the wheel. After performing visual inspection it is necessary to press and hold the brake pedal several times. A slow drop of the brake pedal from under the leg indicates leakage in the system or failure of the brake master cylinder. A different symptom, such as the brake pedal spring-back and rising, will indicate an air lock of the hydraulic brake system [8, 21, 22].

3. Diagnostics mathematical model

The classic calculus of mathematical logic sentences and the classic set theory have one of their generalisations in the so called Boole algebra [6, 10, 12, 13, 14, 15].

The basis for the study of the undertaken issue was the analysis and record of the causes and effects between the HSUH tightness damage in dual terminals of mathematical logic and the set theory.

The diagnostic model of the studied brake system – HSUH (fig. 3) is designated by the following set of material objects:

$$HSUH = \{KP_{HP}, PP_{HP}, LP_{HP}, KP_{HT}, PP_{HT}, LP_{HT}\}$$

where:

KP_{HP} – front brake master cylinder chamber,,

PP_{HP} - right hydraulic sub-circuit (outside the brake master cylinder) of front brakes,

 LP_{HP} – left hydraulic sub-circuit (outside the brake master cylinder) of front brakes,

 KP_{HT} – rear brake master cylinder chamber,

 PP_{HT} – right hydraulic sub-circuit (outside the brake master cylinder) of rear brakes,

 LP_{HT} – left hydraulic sub-circuit (outside the brake master cylinder) of rear brake.

(Fig. 3, next page)

Diagnosis process logical design

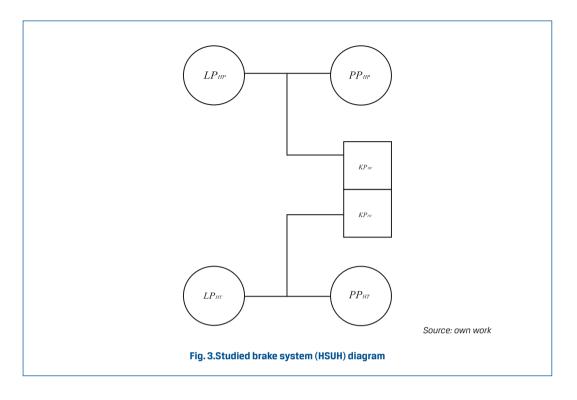
The logical model of the structure of the diagnostics process for the HSUH subsystem tightness/leakage includes the adoption of the following set of mathematical logic sentences describing the leakage conditions of the distinguished material objects:

$$ZL = \{ZW, KPP, KPT, OPP, OPL, OTP, OTL\}$$

where:

ZW – total loss of braking capability (inapplicability of the brake system),

KPP – non-performance of its function by the working chamber of the brake master cylinder of the front axis,



- *KPT* non-performance of its function by the working chamber of the brake master cylinder of the rear axis,
- *OPP* non-performance of its function by the right hydraulic sub-circuit (outside the brake master cylinder) of front axis brakes,
- *OPL* non-performance of its function by the left hydraulic sub-circuit (outside the brake master cylinder) of front axis brakes,
- *OTP* non-performance of its function by the right hydraulic sub-circuit (outside the brake master cylinder) of rear axis brakes,
- *OTL* non-performance of its function by the left hydraulic sub-circuit (outside the brake master cylinder) of rear axis brakes.

The logical model of complete loss of braking capability of the studied HSUH system is described by the following logical equivalence:

 $ZW \Leftrightarrow KPP \land KPT \lor KPP \land OTP \lor KPP \land OTL \lor KPT \land OPP \lor$ $\lor OPP \land OTP \lor OPP \land OTL \lor KPT \land OPL \lor OPL \land OTP \lor OPL \land OTL$ (2)

Equivalence (2) in the technical interpretation informs that the non-performance of its function by the hydraulic brake system takes place only when some of the subsets in the 2-element set do not perform their function:

 $\left\{KP_{HP}, KP_{HT}, PP_{HP}, LP_{HP}, PP_{HT}, LP_{HT}\right\}$

in the following form:

$$\{KP_{HP}, KP_{HT}\}, \{KP_{HP}, PP_{HT}\}, \{KP_{HP}, LP_{HT}\}, \{KP_{HT}, PP_{HP}\}, \{PP_{HP}, PP_{HT}\}, \{PP_{HP}, LP_{HT}\}, \{KP_{HT}, LP_{HP}\}, \{LP_{HP}, PP_{HT}\}, \{LP_{HP}, LP_{HT}\}, LP_{HT}\}, \{LP_{HP}, LP_{HT}\}, LP_$$

Diagnostics for the tightness of the vehicle working brake hydraulic circuit are interesting from the point of view of the HSUH subsystem's user. Therefore, the negation of equivalence (1), on the basis of de Morgan's laws has the following form:

$$\widetilde{ZW} \Leftrightarrow \sim \text{KPP} \land \sim \text{OPP} \land \sim \text{OPL} \land \sim \text{KPT} \land \sim \text{OTP} \land \sim \text{OTL} \quad (3)$$

From the technical point of view it appears that the tightness of the vehicle working brake hydraulic circuit is recorded when all objects in the set are tight:

$$\{KP_{HP}, PP_{HP}, LP_{HP}\},\$$

with any conditions of tightness/leakage of objects in the set:

$$\left\{ K\!P_{HT},PP_{HT},LP_{HT}
ight\}$$

and with tightness of all objects in the set:

$$\left\{KP_{HT}, PP_{HT}, LP_{HT}\right\}$$

with any conditions of tightness/leakage of objects in the set:

$$\{KP_{HP}, PP_{HP}, LP_{HP}\}$$

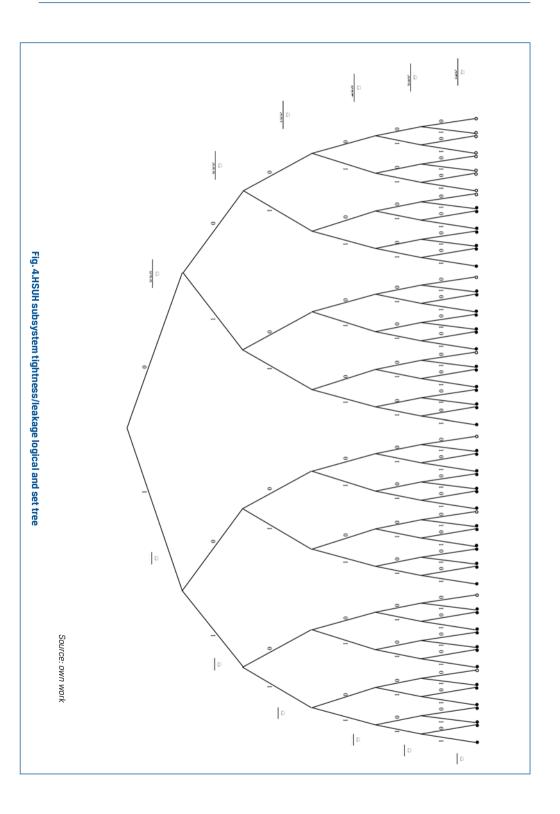
A more in-depth classification of cases of tightness of the studied technical object is given by the logical, canonical, alternative normal form of equivalence (3). In order to simplify the record, it is recommended to abbreviate the logical sentence negation record $p \Leftrightarrow \overline{p}$:

$$\overline{ZW} \Leftrightarrow \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land (KPT \lor \overline{KPT}) \land (OTP \lor \overline{OTP}) \land (OTL \lor \overline{OTL}) \lor \lor (KPP \lor \overline{KPP}) \land (OPP \lor \overline{OPP}) \land (OPL \lor \overline{OPL}) \land \overline{KPP} \land \overline{OPP} \land \overline{OPL} \Leftrightarrow \Leftrightarrow \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land (KPT \land OTP \land OTL \lor KPT \land OTP \land \overline{OTL} \lor KPT \land \overline{OTP} \land OTL \lor \lor VPT \land \overline{OTP} \land \overline{OTL} \lor \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \Leftrightarrow \overline{KPP} \land \overline{OPP} \land \overline{OPL} \Leftrightarrow \overline{KPP} \land \overline{OPP} \land \overline{OPL} \Leftrightarrow \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPL} \land \overline{KPT} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OPP} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OTP} \land \overline{OTP} \land \overline{OTL} \lor \overline{KPP} \land \overline{OPP} \land \overline{OTP} \land \overline{OTP} \land \overline{OTP} \land \overline{OTL}$$

Equivalence (4) is coded in an alternative form of only 15 canonical elementary conjunctions. The theorem of logical disjunction in the space of canonical elementary ratios with 6 variables demonstrates that the canonical alternative form of normal equivalence (1) is an alternative for 49 canonical elementary ratios (see the logical tree in fig. 4).

15 cases of classification of the diagnostics process for the tightness of the HSUH subsystem working brake hydraulic circuit are illustrated in fig. 4 by the logical tree paths marked with the "o" symbol.

They confirm the technical interpretation formulated above.



Diagnosis process set model

The set model of the studied *HSUH* subsystem includes consideration of the following damage sets H= set of all damage of the $HSUH = (KP_{HP}, PP_{HP}, LP_{HP}, KP_{HT}, PP_{HT}, LP_{HT})$ system's working brake hydraulic circuit,

NH – set of all leaks of HSUH, NKP_{HP} – set of all leaks of KP_{HP} , NPP_{HP} – set of all leaks of PP_{HP} , NLP_{HP} – set of all leaks of LP_{HP} , NKP_{HT} – set of all leaks of KP_{HT} , NPP_{HT} – set of all leaks of PP_{HT} , NLP_{HT} – set of all leaks of LP_{HT} .

In the formal language of the set theory, the model structures of the diagnostics process for the tightness/leakage of the *HSUH* subsystem working brake hydraulic circuit are described by the following set equations:

$$NH = (NKP_{HP} \cup NPP_{HP} \cup NLP_{HP}) \cap (NKP_{HT} \cup NPP_{HT} \cup NLP_{HT})$$

and

$$NH' = NKP'_{HP} \cap NPP'_{HP} \cap NLP'_{HP} \cup NKP'_{HT} \cap NPP'_{HT} \cap NLP'_{HT}$$
(5)

In accordance with the principle of duality between the structures of mathematical logic and the set theory (negation, conjunction and alternative of sentences are respectively dual in relation to the notions of complementation, ratio and union of sets), it is possible to skip the in-depth analytical deliberations and in case of complementation of the set of all cases of leakage, to formulate a lesser number of canonical ratios of elementary sets (see equation (5))

$$NH' = NKP'_{HP} \cap NPP'_{HP} \cap NLP'_{HP} \cap NKP_{HT} \cap NPP_{HT} \cap NLP_{HT} \cup NKP'_{HP} \cap NPP'_{HP} \cap NLP'_{HP} \cap NKP_{HT} \cap NPP'_{HP} \cap NLP'_{HP} \cap NL$$

The above equation of set algebra (6) illustrates an analogous technical interpretation to the one formulated on the basis of the logical equivalence (4).

4. Summary

Designing of a diagnostics device for braking mechanism in vehicles is a very important issue. Especially if it is related to the aspect of diagnostics used in automation of braking processes, which widely apply electronic and IT tools.

Diagnostics is acting on the symptoms of the technical object's condition, while in case of its failure – acting on its syndromes. One of the symptoms - syndromes of vehicle hydraulic brake system inapplicability is its leakage.

An important stage in the process of designing diagnostic systems for machines, including braking mechanisms is the modelling of brake elements, braking process, brake operation, its degradation and damage.

The performed analysis of the structure of the process of diagnostics of the vehicle hydraulic brake system leakage allowed for the specification of decisional variables and their relations in formal languages of mathematical logics and the set theory. On this basis the mathematical model for the process of diagnostics of the hydraulic circuit leakage between the double-sequence brake master cylinder (divided into two working chambers) and the left and right hydraulic circuit of front and rear axis brakes.

The obtained results of structuralisation can be applied in the automation and robotics of the process of diagnostics of the vehicle hydraulic brake system tightness/leakage.

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